PG – 217

II Semester M.Sc. Degree Examination, June 2016
(CBCS)
Mathematics
M 202T : COMPLEX ANALYSIS

Time : 3 Hours   Max. Marks : 70

Instruction : *Answer **any five full** questions.*

1. a) Let f(z) = u(x, y) + iv(x, y) be an analytic function. Then show that real and imaginary parts of an analytic functions are Harmonic. Also evaluate $\int_c \frac{e^z}{z(z-1)(z-2)} dz$ where $c : |z| = 2$.

   b) Define conformal mapping. Discuss the transformation $w = e^z$.

   c) State and prove the generalization of Cauchy's integral formula and use it to evaluate $\int_c \frac{dz}{z^4 - 1}$, $c : |z| = 2$. **(4+4+6)**

BMSCW

2. a) State and prove Cauchy's theorem for a disk.

   b) If f(z) is analytic in a region C of complex plane, then prove that the following statements are equivalent :

      i) $f^n(a) = 0$, ∀ n = 0, 1, 2, ... at a point 'a' in C.

      ii) f(z) = 0 in a neighbourhood K of a point 'a' in C.

      iii) f(z) = 0 in C. **(6+8)**

3. a) Define radius of convergence. Let $f(z) = \sum_{n=0}^{\infty} a_n (z-a)^n$ in $\{|z-a| < R\}$ where R is radius of convergence of the power series. Then prove that the Taylor's expansion of f(z) in the neighbourhood of a point 'a' is exactly the given power series.

   b) Find the radius of convergence of

      i) $\sum_{n=0}^{\infty} \frac{(2n)^n}{n!}$ ii) $\frac{1}{2}z + \frac{1.3}{2.5}z^2 + \frac{1.3.5}{2.5.8}z^3 + ...$

   c) Find the power series expansion of $f(z) = \frac{z+1}{z(z^2 - 4z + 3)}$, in the regions.

      i) $0 < |z| < 1$ ii) $1 < |z| < 3$. **(6+4+4)**

P.T.O.

4. a) State and prove Laurent's theorem.

   b) Let f(z) be analytic function having an isolated singularity at z = a. If $|f(z)|$ is bounded in a neighbourhood $\{0<|z-a|<r\}$ then prove that f(z) has a removable singularity at z = a.

   c) State and prove open mapping theorem. **(6+4+4)**

5. a) State and prove Cauchy's residue theorem.

   b) Evaluate **any two** of the following :

   i) $\int_0^{2\pi} \frac{d\theta}{1-2p\cos\theta+p^2}$ ii) $\int_{-\infty}^{\infty} \frac{\sin x\, dx}{x(x^2-2x+2)}$ iii) $\int_0^{\infty} \frac{dx}{(x^2+1)^2}$

   (4+10)

6. a) Let f(z) analytic inside a simple closed contour γ and let f(z) be continuous on γ. If f(z) has no zeros on γ then prove that f(z) has only a finite number of zeros in the interior of γ.

   b) Prove that all the roots of $z^7 - 5z^3 + 12 = 0$ lie between the circles $|z| = 1$ and $|z| = 2$.

   c) Let f(z) be an analytic in the disc $\{|z|<R\}$, R > 0, then show that M(r) is a monotonically increasing function of R in [0, R] unless f(z) is not constant. **(5+4+5)**

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7. a) State and prove Hadamard's three circle theorem and prove that log M(r) is a convex function of log r.

   b) State and prove Weierstrass factorization theorem. **(7+7)**

8. a) Define Harmonic function. State and prove the mean value property for harmonic functions.

   b) Derive the Poisson's integral formula with standard notation. **(7+7)**
